THE LANGUAGE OF MATHEMATICS AND FLUENCY

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Abstract

The language of mathematics and fluency

The problem of mathematics avoidance in the United States has a massive effect on how students can compete with students of other nations in a world economy. This problem may have it roots in the attitudes toward and difficulties in learning a second language, prevalent in American culture.

This project used a conceptual methodology to consider the relationship of mathematics as a language and its relationship to mathematics avoidance and anxiety. Specifically, philosophical inquiry was used to probe the concept of mathematics as a language and unpack ideas surrounding the definition of fluency. The study compared the philosophies of the original authors of mathematic language and contemporary philosophy of linguistics to further understand the dynamics and assumptions behind the idea of fluency in language and its relationship to the language of mathematics.

The study concluded that the current definition of fluency in mathematics was not adequate to account for the necessities of learning the meta-language of mathematics. Fluency should be defined as the ability to express abstract thought, principally communicated through discourse, and create meaning through the use of grammar and prior knowledge.

The lack of fluency and elements of mathematical discourse at the elementary, middle school and junior high school level contributes to the prevalence of mathematics avoidance.

Table of Contents

List of Figures

Introduction

The average mathematics teacher seems to have a few students that struggle through their classes. Maddy, Nikki and Conner, students in an 8th grade mathematics class, are no different. They always seem to have little confidence in their own abilities and fight learning new concepts. The teacher walks through the problems step by step with them after class and watches them perform each step flawlessly, but when asked to perform the same steps independently they balk, often commenting, "I don't get it", or "I'm just not good at math." Maddy, Nikki and Conner are like many middle school students that would rather avoid math class altogether. These students and many others are suffering from mathematics avoidance and anxiety.

Mathematics avoidance and anxiety, is most commonly defined as feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in life and academic situations (Richardson, & Suinn, 1972). The attitudes and perceptions of students have a direct effect on student learning. These perceptions are shaped by many factors; parental and teacher perceptions, prior achievement, and perceptions of difficulty (Eccles & Jacobs, 1986). These factors show that mathematics avoidance and anxiety are socially constructed, and lead us to question whether it is only prevalent in the United States.

A cross cultural study was conducted in 2003 to determine the prevalence of mathematics anxiety, although mathematics anxiety was found present in most countries. Asian countries including China and Japan demonstrated higher levels of mathematics anxiety than western countries, but demonstrated higher achievement scores (Lee, 2009). This study seems to discount the prevalence of anxiety as a factor in the achievement gap problem; perhaps it is not anxiety but the avoidance of mathematics in general.

In the United States, seventy-five percent of Americans stop studying mathematics before they have completed the education requirements for their career or job (Scarpello, 2007). A recent study involving first and fifth grade students from the United States, Japan, and China, determined that the cultural perception of mathematics was the major contributing factor in the achievement gap (Stevenson, Lee, & Stigler, 1986). Students in China and Japan spend twice as much time learning math in school and at home as students in the United States. Teachers and parents in the United States believe that more time should be spent in the instruction of reading and writing than mathematics (Stevenson, Lee, & Stigler, 1986). Mathematics avoidance may be a byproduct of this perception making it culturally acceptable to avoid taking further mathematics courses.

If we assume that mathematics is a language then we can draw some conclusions about mathematics avoidance and anxiety by comparing it to the behavior of students acquiring a second language. The acquisition of a second language is affected by attitudinal factors that encourage a student to learn and make the student open to new languages (Krashen, 1981). The focus of elementary education in the United States is literacy in the English language and does not promote the learning of a second language. This may account for the proportionately lower number of bilingual students and the development of negative attitudinal factors towards the learning of new languages and mathematics (Stevenson, Lee, & Stigler 1986). This may be the underlying cause of mathematics avoidance and anxiety.

The key to combating mathematics avoidance and anxiety is to understand its origins and the cultural settings which create these negative attitudes. A closer look at the language of mathematics and its acquisition may give us a deeper understanding into the cultural causes of mathematics avoidance.

Literature Review

The goal of this literature review is to provide a background with which to develop a deeper understanding of the philosophies and common assumptions which surround socially constructed perceptions of mathematics and the language of mathematics. The review will consist of three main areas of concentration: mathematics avoidance and anxiety, the language of mathematics, and current trends and ideas surrounding the avoidance of learning a second language. First, what is mathematics avoidance and anxiety, and what does current research tell us about mathematics avoidance and anxiety?

Mathematics avoidance and anxiety

Mathematics anxiety is defined as feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems (Richardson, & Suinn, 1972). Mathematics anxiety is prevalent in many countries but its effects are mitigated by cultural values and norms.

The 2003 Program for International Student Assessment (PISA) project sampled 250,000 fifteen year old students across forty-one countries, to evaluate the perceptions of self-concept, self-efficacy, anxiety, and mathematic achievement (Lee, 2009). The PISA study defined self-concept as one's perception of the self that is continually evaluated and reinforced by personal inferences about oneself (Lee, 2009). Self-efficacy is defined as one's conviction or belief about his or her capability to successfully produce a desirable outcome (Lee, 2009). The study concluded that in all cultures there is a strong correlation between mathematics anxiety and performance scores with peers within that culture. Students having high levels of mathematics anxiety consistently score lower than those with lower levels of mathematics anxiety. The magnitude of this effect was strongly mitigated by cultural values and norms (Lee, 2009).

Japan, Korea, China, and Thailand showed high levels of anxiety and low self efficacy, but their mathematics achievement scores were consistently above average. These countries have high academic standards and a high level of normative testing which may have a direct impact on levels of anxiety. Parents hold high expectations of student performance and little confidence in the performance of the educational system (Lee, 2009).

Western European countries exhibited high self-efficacy and low anxiety scores with achievement scores slightly above average. These countries tended to have a more relaxed school environment which accounts for the lower levels of anxiety (Lee, 2009).

The United States on the other hand scored high in self-concept, self-efficacy, and moderately high in anxiety and made slightly below average achievement scores (Lee, 2009). The United States student experience aligned itself closer to those of Western Europe (Lee, 2009).

The study confirmed that mathematics anxiety has an effect on mathematics achievement, but this effect is mitigated by educational programs, cultural norms and values (Lee, 2009). The study however was unable to pinpoint exactly which cultural norms, values, or educational programs affected achievement scores. Since anxiety does not seem to have a major effect on academic achievement it is necessary to take a closer look at the factors that contribute to mathematics avoidance.

Eccles and Jacobs (1986) conducted a two-year longitudinal study of 250 seventh through ninth grade students, which included the parents and teachers. The goal of the study was to identify the social factors that shape gender biases and predict mathematics achievement and participation. The study identified past performance, the mothers' perception of task difficulty for the child and the teacher's estimates of the child's mathematics ability as the three major factors in mathematics avoidance (Eccles, & Jacobs, 1986). These findings were universal for boys and girls. Eccles and Jacobs concluded that student attitudes toward math and plans to continue to take math classes were strongly influenced by the parent's perceptions of the difficulty of mathematics for their child, and their own attitudes about the value of mathematics.

Susan Stodolsky conducted an observational study of fifth grade mathematics and social studies classes to determine how teacher instructional methods shaped negative attitudes towards mathematics. Stodolsky found that mathematics instruction often varied vastly from instruction in other content areas (Stodolsky, 1985). These instructional differences tell the student that there is something different about learning mathematics and adds to the perceptions of difficulty and ability to learn. These perceptions are profoundly different when students considered other content matter.

Many high school students and adults in the United States do not like mathematics and perceive it as difficult. Some become anxious when faced with mathematical problems. The idea that you are or are not good at math is readily accepted among adults, whereas such distinctions are not made in other fields such as reading, English, or social studies" (Stodolsky, 1985. p. 131).

Stodolsky determined that the effects of instructional forms and procedures not only effect achievement, but developed attitudes and perceptions about how learning occurs within the subject matter.

Stevenson, Lee, and Stigler (1986) determined that the social factors contribute greatly to the mathematics achievement gap between Asian countries and the United States. Stevenson, Lee and Stigler conducted a study of mathematic achievement between Japan, China, and the United States and determined that the gap in mathematic achievement starts in the elementary school. The study included 480 students from each country and attempted to map the progression of the achievement gap and identify the social causes of the gap. Most students entered elementary school with relatively the same mathematical achievement, but by the fifth grade there is a marked separation represented in Figure 1 (Stevenson, Lee, & Stigler 1986).

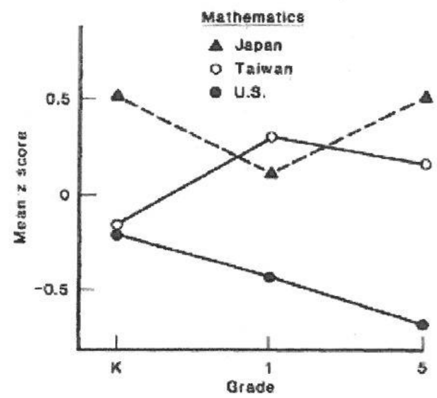

Figure 1 Children's performance on mathematics test

Stevenson, Lee, and Stigler further determined that the major cause for the separation was the time spent teaching and practicing mathematics.

> The lack of time spent teaching mathematics may be a reflection of the view of American parents and teachers that education in elementary school is synonymous with learning to read. Large amounts of time are devoted to reading instruction, and if changes were to be made in the curriculum, both parents and teachers agreed that even greater proportions of time should be devoted to reading. Mathematics and science play a small role in Americans' conception of elementary education. (Stevenson, Lee, & Stigler, 1986. p. 698)

Only one American teacher advocated the desire to spend more time teaching math (Stevenson, Lee, & Stigler 1986). Most Americans will say that mathematics is important, but when it comes down to implementation it seems to be relegated to a secondary role in education.

The language of Mathematics

The language of Mathematics bears further study to answer the questions: Is mathematics a language? If mathematics is a language how, does that impact the way we teach mathematics? This section reviews research focused on the view that mathematics should be seen as a language and thus should be taught so.

Dehaene, Spelke, Pinel, Stanescu, and Tsivkin (1999) conducted a study using brain-imaging to determine if mathematics was linked to language thinking centers. It was discovered that the interpretation of mathematic problems fell on areas of the brain normally associated with language. However, the areas of the brain activated when the subject was actually doing approximations were in the area normally associated with visuo-spatial and analogical mental transformations (Dehaene et al. 1999). The study was conducted using bilingual participants, which were presented math problems randomly in both languages; the subject was then required to solve the problems as fast as possible. The study showed that interpretation of math problems were independent of the language in which it was presented. Dehaene et al. (1999) concluded that exact calculations are language dependent and approximation relies on the nonverbal visuo-spatial cerebral networks. The results suggest that the teaching of advanced mathematical facts gives rise to a language independent conceptualization (Dehaene et al. 1999). Dehaene et al. states "Many domains of mathematics, such as calculus, also may depend critically on the invention of an appropriate mathematical language" (1999, p. 973). This leads us to the idea that language of mathematics is separate from the verbal language spoken by the individual.

O'Halloran (2005) approaches the language of mathematics from a different direction implying that because its form is similar to formal language models, it must be considered a language. O'Halloran uses the models suggested by Michael Halliday (2003) to prove that mathematics is a language, specifically the idea of multisemiotic discourse and grammar. The concept that mathematics has specific symbols and metaphorical constructions that can only be done using these symbols sets it aside as a language. The second point O'Halloran states is the specific grammar, the order and rules implied by mathematics constitute it as a language.

"In Literacy in the Language of Mathematics" James Bullock states "If mathematics consisted only of new words and symbols, it could properly be considered as an extension of existing language. The reason mathematics is a new and separate language is that it also has its own syntax and grammar." (1994, p. 736). He goes on to say that many of the metaphors constructed in mathematics can only be described using mathematics much like many spoken languages. Bullock is an advocate of literacy and states that it is not enough for students to translate/answer questions using a process but be able converse mathematically.

> Mathematics is not a way of hanging numbers on things so that quantitative answers to ordinary questions can be obtained. It is a language that allows one to think about extra ordinary questions. Saying that the earth has a round shape means only that it has no edges. This non-mathematical picture is not simply "qualitative", "verbal", or "intuitive"; it is primitive and empty. If we wish to construct a meaningful metaphor about the shape of the earth, we must use the language of shapes, which is mathematics. (Bullock, 1994. p.737)

Bullock (1994) concludes that the reason mathematics came into being was the inability of existing language to deal with the subject matter.

The process of acquiring a new language relies on the development of what is correct or proper use of a language. Yackel and Cobb address this issue by defining sociomathematical norms and addressing how they are developed in the classroom. Sociomathematical norms are normative aspects of mathematical discussions that are specific to students' mathematical activity (Yackel, & Cobb, 1996). The normative understandings are developed through discourses between the student and teacher; these norms are continually regenerated and modified with each interaction much as we develop fluency in a spoken language (Yackel, & Cobb, 1996).

The avoidance of acquiring a second language

In this section I will review current literature on the avoidance of obtaining a second language other than the initial language learned. There are many social constructs which cause one language, to be dominant over another.

Languages In Contact: Findings and Problems by Ureil Weinreich (1968) defines ten reasons for one language to be dominant over another all of which are nonlinguistic in nature; geographic areas, indigenousness, cultural or ethnic groups, religion, race, sex, age, social status, occupation, and rural vs. urban population. These factors play a role in how and why a culture may down play the acquisition of a second language. These may seem strait forward but there are many subtleties for example occupational may include the keeping of trade secrets. Social status seems to play a bigger role in learning a second language; i.e. it is more acceptable for students of low socioeconomic status to avoid a new language as it is perceived as not necessary for success. The high socioeconomic status students see a direct benefit in learning second language as their parents are an immediate example of why it is necessary (Weinreich, 1968).

Weinreich (1994) states that perceived difficulty and level of fluency can influence and deter a new language learner. Students that believe the new language to be too abstract or difficult will avoid practice, and everyday use of the language. Students that do not become fluent in simple language tasks, do not feel comfortable enough to use the language on a regular basis, avoiding opportunities to learn more complicated or refined uses of the language.

Cates and Rhymer (2003) define fluency in mathematics as the ability to perform a behavior correctly, quickly, and with minimal effort. Cates and Rhymer (2003) conducted a study to measure fluency and anxiety in mathematics by testing accuracy and fluency in the first two stages of the learning hierarchy. The learning hierarchy for learning skills consists of four levels, acquisition, fluency, generalization, and adaption. The goal of the acquisition stage is produce the correct response in the absence of a time limit. The fluency stage requires the student to obtain a solution as quickly as possible. When a student is able to perform the behavior under different conditions then they have entered the generalization stage. The adaption stage is where a student begins to synthesize new ways to use the behavior (Cates, & Rhymer, 2003). Cates and Rhymer (2003) discovered that the students which exhibited high levels of mathematics anxiety scored the same as students without, at the acquisition level of learning. A separation in accuracy did not show until students were tested above their level of fluency, consequently resulting higher levels of anxiety in all students tested. Cates and Rhymer concluded "When students' progress beyond the initial acquisition stages and over learn material to the extent of being fluent or automatic, these students may be less likely to exhibit higher mathematics anxiety levels for basic skills" (2003, p.31). This study also concluded that educators should consider the rate in which they increase difficulty until students have some degree of fluency (Cates, & Rhymer, 2003).

A review of the literature tells us that mathematics avoidance, not anxiety is the major cause of the present gap in mathematic achievement. The causes of avoidance are socially constructed and may be linked to cultural values. Individual teachers do not have the capability to make sweeping changes in society but they do have some effect on the student's perception of math.

The review also tells us that mathematics is a language and that the avoidance of mathematics may be attributed to factors typically attributed to learning a new language. The single factor that a teacher may be able to change is developing fluency and changing perceptions of difficulty in mathematics if we treat it as a language.

Research Question

The literature indicates that mathematics is a form of language and the processes required to learn this language plays a role in mathematics anxiety and avoidance behavior in the United States. The literature indicates that fluency plays a major role in learning a new language.

The Literature reveals that there is a disconnect in which the way fluency is defined in linguistics and how it is defined in mathematics. The conflict in definitions leads me to my primary research question.

What does fluency mean in the language of Mathematics?

This project was also focused in the following secondary question.

> Should the development of fluency in mathematics play a more prevalent role in mathematics education as a means of increasing learning and combating mathematics avoidance and anxiety?

Methodology

Method and Rationale

This project uses a conceptual methodology to consider the relationship of mathematics as a language and its relationship to mathematics avoidance and anxiety. Specifically, philosophical inquiry was used to probe the concept of mathematics as a language and unpack the ideas surrounding the definition of fluency. The study compared the philosophies of the original authors of mathematics language and contemporary philosophies of linguistics to further understand the dynamics and assumptions behind the idea of fluency in language and its relationship to the language of mathematics.

Sample and Instrumentation

The data for the study incorporates the philosophies of the original developers of mathematics and significant contemporary linguistic philosophies. The study will expand upon the idea that mathematics is a language and to develop the assumptions around fluency. These philosophers include:

- Michael Halliday
- Plato
- Rene Descartes

Michael Halliday is a leading linguistic theorist focusing on how humans learn and apply languages. Plato's philosophy gave rise to inductive reasoning and his ideas on the purposes of education and learning in relation to the dialectic, shed light the language of mathematics. Rene Descartes developed the idea of using symbols and variables in analytical geometry and creating the Cartesian coordinate system.

Analysis/Validity

This study seeks to compare the philosophies surrounding the structure of language and mathematics, and define fluency and the role it plays in the acquisition of new language learning and mathematics.

Through the philosophy of Halliday I identify what fluency means in regards to linguistics, and then seek to compare and contrast this definition with the philosophies of Plato and Descartes. Through these comparisons I determine a more accurate definition of what fluency in mathematics is, and the role and value of fluency in learning mathematics.

As a mathematics teacher, with a mathematics degree I am well versed in the technical language of mathematics, and have a tendency to have my own ideas of what fluency in mathematics should be. I used an independent reader and reviewer to verify my interpretation of the various philosophical writings. I purposely chose philosophers that are diametrically opposed in their learning philosophies to ensure I get a rounded a more rounded view of what fluency should mean.

Data

Halliday language and Fluency

Halliday a noted linguist, developed the idea of systematic functional linguistics, this treats language as foundational for the building of human experience. *On language and linguistics* and *The Language of Early Childhood* are used to explore how children learn new languages, and to solidify what fluency should look like from a linguistic view.

Halliday defines language as a system of meanings, that a language both creates and exchanges meaning.

> A language is a system of meaning-a semiotic system. "Semiotic" means having to do with meaning (semiosis); so a system of meaning is one by which meaning is created and meanings are exchanged. A language is almost certainly the most complicated semiotic system we have; it is also a very fuzzy one, both in the sense that its own limits are unclear and in the sense that its internal organization is full of indeterminacy. We could then think of a semiotic system as being of a fourth order of complexity, being semiotic and social and biological and physical: meaning is socially constructed, biologically activated and exchanged through physical channels (Halliday, 2003, p.2).

This definition of language states that language is not just a tool used for communication but a complex semiotic system for learning. This system is not limited as a device to exchange ideas and meaning, it also serves as a means to create new ideas and meaning. The concept that language both exchanges and creates meaning is foundational to Halliday's philosophy on language and learning. Halliday believes that it is this process of communicating and creating meaning that allows us to learn using language.

Halliday divides the process of learning language into three facets learning language, learning through language, and learning about language. Through these three facets he traces how children acquire language.

> (Halliday, 2004) There are, I think, three facets to language development: learning language, learning through language, and learning about language.

> First, then: "learning language". A child starts learning language from the moment he is born; newborn babies are very attentive listeners. No doubt, in fact, the baby has already started learning language before he was born, picking up the rhythms of speech from their source in his mother's diaphragm. But from birth onwards he is actively involved in communication, exchanging signals with the other human beings around him. For this purpose he has to construct a language; and we are now beginning to understand something of how he does it (Halliday, 2004, p.308).

These three facets are building blocks and each has distinctive characteristics, they do not necessarily occur individually but may have some overlap. The first facet learning language is represented by what Halliday refers to as "protolanguage."

Halliday describes "protolanguage" as the beginning language where we are unable to communicate complete ideas or thoughts verbally.

> Children's first language-like semiotic system, which I labeled "protolanguage" when I observed and described it thirty years ago, begins as a collection of simple signs. These signs soon come to be organized into minimal systems, like ' I want'/'I don't want'; and these show the beginning of further organization in clusters, on a functional basis; but they are not yet combined, nor are they yet layered or uncoupled (Halliday, 2003. p. 7).

This "protolanguage" is incapable of creating meaning at this point because it lacks the structure and order given by grammar. Halliday states that "Meaning was not made of words; it was construed in grammar as much as in vocabulary, and even if we could assess the quantity; of words the learners knew it would give little indication of what they could do in the language" (2003, p.8). Halliday believes that it is this lack of grammar as the separating factor between learning a language and learning through language.

> Protolanguage has s semantics and a phonology, but no level of grammar between the two. In other words, it is not yet stratified. The grammar emerges later, as the child moves from child tongue to mother tongue during the course of the second year of life (Halliday, 2003, p.13).

Although the protolanguage has an order it lacks the ability to connect meanings together to create new meaning, this is the role of grammar.

The emergence of grammar signals the emergence of the second facet of learning, "learning through language."

> "Learning through language" refers to language in the construction of reality: how we use language to build up a picture of the world in which we live. As a child begins the transition from protolanguage to language- from child tongue to mother tongue-he comes to make a rather systematic distinction between two basic functions of language, which I have referred to as the "pragmatic" and the "mathetic", the doing function and the learning function (Halliday, 2004, p. 317).

This learning through language as defined by Halliday is how we learn to describe the physical world around us. More profoundly Halliday extends this to the ability further to describe hypothetical or purely abstract ideas.

> The move into grammar is the step from primary consciousness to higher order consciousness. When our primary semiotic evolved into a higher-order semiotic (that is, when protolanguage evolved into language), a space was created in which meanings could be organized in their own terms, as purely abstract network of interrelations. By "purely abstract" I mean not interfacing directly with the ecosocial environment (Halliday, 2003, p.14).

Grammar according to Halliday is what allows us to create meaning in the world around us. Halliday's pivotal philosophy about language is based predominantly on this idea.

Halliday believes that language is the way humans make sense of their existence and it is through language that we interpret the world around us.

> Most obviously, perhaps, when we watch small children interacting with objects around them we can see that they are using language to construe a theoretical model of their experience (Halliday, 2003, p. 15).

This belief extends not just to the physical world but to how we interpret our experiences as well giving definition through the creation of meaning which allows us to interact with others on a higher level.

> We should stress, I think, that the grammar is not merely annotating experience; it is construing experience-theorizing it, in the form that we call "understanding". But from the start, in the evolution of language out of protolanguage, this "construing" function has been combined with another mode of meaning, that of

> enacting: acting out the interpersonal encounters that are essential to our survival (Halliday, 2003, p. 16).

Halliday believes that this concept is essential in understanding language, its purpose and how we learn as human beings. He also believes that this ability is an essential component in being human (Halliday, 2003). Learning about language is the final facet in the learning process.

Halliday asserts that we reach the final facet of learning a language when we are able to start using language for the sole purpose of creating meaning.

> My third heading was "learning about language"; in other words, coming to understand the nature and functions of language itself. In one sense, every human being knows about language simply because he talks and listens, but this is unconscious understanding, in the same way that our knowledge of language is unconscious knowledge it is knowledge stored in the gut, so to speak (which is where many cultures locate true understanding), rather than knowledge stored in the head (Halliday, 2004, p. 322).

The idea that when we can understand and learn the nature of the language we truly begin to be able to use the language to its fullest. This third facet is characterized by the ability to put the language into a written form. Halliday states "He also has to learn that writing maps on to the words and structures that by this time are already embedded deeply in his unconscious knowledge of the world" (2004, p. 325). The reason Halliday believes that this represents the third facet is that it puts language into a permanent form, subject to interpretation by others. "Writing puts language in chains; it freezes it, so that it becomes a thing to be reflected on" (Halliday, 2003, p. 132). The learning about language seems to be the final stage in language learning, and implies true fluency. Halliday believes that the written is the final demonstration of this fluency (Halliday, 2004).

The way we use language to facilitate learning is of special interest to Halliday, he believes that it is through the process of communication that all learning exists.

Halliday states, "I sometimes ask teachers about this question: whether there are things in the curriculum they consider best learnt through talking and listening, and other things best learnt through reading and writing" (Halliday, 2003, p. 134). He questions the common process of reading and writing as the main source of learning and believes we should focus more on the meanings and how they are created rather than the function and processes.

> For educational purposes we need a grammar that is functional rather than formal, semantic rather than syntactic in focus, oriented towards discourse rather than towards sentences, and represents language as a flexible resource rather than a rigid set of rules. (Halliday, 2004, p. 323)

Learning should be tailored to the students' needs and language learning should not be treated as just one subject but that it encompasses all learning. "When children learn language, they are not simply engaging in one kind of learning among many; rather they are learning the foundation of learning itself" (Halliday, 2004, p. 327).

Plato on learning, inductive logic, education, and the dialectic

Plato developed the logical process of inductive reasoning, the proving of an idea or assumption through the use of examples. Using the writings of Plato; *Euthyphro, Phaedo, Menos, and The Republic* I will explore the Plato's views on learning, inductive logic, education, and the dialectic, to understand how Plato might have viewed or defined fluency.

To understand Plato's role in the development of mathematics and the idea of fluency we must understand his basic ideas about learning and where these ideas came from. Plato's two basic tenants of learning, the learning process is really only recollection of what we already know and the platonic ideas that all truth can only be discovered through reasoning.

The idea that learning as a function of recalling that which we already know is based in Plato's belief in Greek religion and its ideas that the soul is recycled through many life times. "All Knowledge is recollection based on previous experience of 'what the soul has learned' " (Plato, 2011a, p. 6); this principle still has some merit in that we can only learn if we recall what we already know and can apply that knowledge to new situations. Plato repeatedly relies on this argument in several situations throughout his writings.

> That our learning is simply recollection- that argument, also, it is sound, proves that we must have learned what we now recollect at some previous time (Plato, 2011a, p. 78-79).
>
> "I can give you an excellent reason," said Cebes, "when people are asked something, if the question is well put, they themselves explain everything and yet if they hadn't got knowledge and a right account of the matter stored away inside them, they couldn't do that; and if you next take them to the figures of geometry or something else of that sort, it is then as clear as could possibly be that this is the case" (Plato, 2011a, p. 79).

It is this philosophy that shapes his thoughts on many matters of philosophy and mathematics. The second philosophy of Plato that impacts his ideas about education and learning is the philosophy that all learning or thought should only be done through reasoning alone.

Plato believed that it was only through reasoning that we have the power to find the truth of a matter.

> "Now how about the acquisition of wisdom? Is the body a hindrance, or is it not, if you use it as an accessory in the search? What I mean is, do sight and hearing provide men with any true knowledge, or are even the poets always trying to tell us something like this, that nothing that we hear or see is accurate? And yet if

> these bodily senses are not accurate or reliable, the others are hardly likely to be-for all the others are inferior, I suppose, to these. Don't you think so?"
> "I do indeed." He said.
> "When, then, does the soul attain to truth?" he went on. "when it tries to investigate anything with the help of the body, the body quite clearly deceives it....So it is only through reasoning, if at all, that any part of reality can be plainly understood" (Plato 2011a, p. 71).

Plato's belief that the senses tend to cloud reason with emotion and desires therefore we should forgo the indulgences of the body and seek the truth of a matter using our minds. This idea extends throughout Plato's works and plays a central role in how he views education, and his development of inductive logic.

Inductive logic

Plato's proofs are based on the principle of inductive logic. Plato uses examples to identify the traits or characteristics or what is known extends those characteristics to prove his educated hypothesis. Plato uses inductive logic in almost every discourse to demonstrate his philosophy.

> "And does it not follow from all this that the recollection can be caused by what is like or by what is unlike"
> "Yes indeed."
> "But when you are reminded of something by what is like it, are you not bound also to notice whether this similar thing falls short or not in any way in its resemblance to the thing of which you have been reminded?"
> "Necessarily," he said.
> "Now consider whether this is true. We say, I think, that there is a thing which is Equal-I don't mean a particular piece of wood that is like another, or a stone that is like another, or anything of that sort, but something over and above all these, the Equal itself. Are we to agree that there is such a thing, or not?"
> "Now when we have to do with the pieces of wood and the equal things we were talking about just now, do they seem to us to be equal in the same way as that which is essentially and perfectly equal, or do they, perhaps, fall short of that in point of resemblance to what is equal?"
> "They fall short a great deal,' he said.
> Then we agree that when a man sees a thing, and tells himself that 'the thing I am now looking at wants to be like some other thing,' but that if falls short and cannot be like that-that it is, in fact, inferior-The man who gets this notion must I

> suppose, have previous knowledge of that thing to which he says that he sees a real but imperfect resemblance."
> "Then we must have had knowledge of the Equal before that time when we first saw the things that are "equal" and conceived that idea that all these things were trying to be like the Equal, but fell short."
> "But we also agree that we derived the conception from no other source" (Plato, 2011a, p.79- 81).

In this example he uses inductive logic to prove that learning is only the recalling that which we already know. This inductive logic plays a central role in platonic philosophy and is heavily influenced by Plato's belief that it is only through higher consciousness or reasoning that we can find truth. It is these principles that lead to his ideas surrounding education.

Education

In *The Republic* Plato extends his philosophy of higher thinking and learning to education. Plato's ultimate goal for education is to develop higher thinking, or reasoning for the sake of pure knowledge or theoretical thought.

> We can, then, properly lay it down that arithmetic shall be a subject for study by those who are to hold positions of responsibility in our state; and we shall ask them not to be amateurish in their approach to it, but to pursue it till they come to understand, by pure thought, the nature of numbers-they aren't concerned with its usefulness for commercial transactions, as if they were merchants or shopkeepers, but for war and for the easier conversion of the soul from the world of becoming to that of reality and truth (Plato, 2011b, p. 283-284).

Plato delineates a curriculum for the sole purpose of posing problems for this higher thinking. Plato believes that mathematics should be the center piece of his educational system because the various disciplines can be extended to the theoretical and promote this higher thinking.

> 'you know' I said, 'now that we have mentioned the study of arithmetic it occurs to me what a subtle and widely useful instrument it is for our purpose, if one

studies it for the sake of knowledge and not for commercial ends. (Plato, 2011b, p. 284)

It draws the mind upwards and forces it to argue about numbers in themselves, and will not be put off by attempts to confine the argument to collections of visible or tangible objects (Plato, 2011b, p.284)

Another point- have you noticed how those who are naturally good at calculation are nearly always quick at learning anything else, and how the slow witted, if trained and practiced in calculation, always make progress and improve in speed even if they get no other benefit (Plato, 2011b, p. 284)

They talk about "squaring" and "applying" and "adding" and so on, as if they were doing something and their reasoning had a practical end, and the subject were not in fact, pursued for the sake of knowledge. (Plato, 2011b, p.285)

'Its usefulness for war, which you have already mentioned, ' I replied; ' and there is a certain facility for learning all other subjects in which we know that those who have studied geometry lead the field.' (Plato, 2011b, p.285)

The right thing is to proceed from the second dimension to third, which brings us, I suppose, to cubes and other three-dimensional figures. (Plato, 2011b, p. 287)

Plato chooses arithmetic, geometry, solid geometry, astronomy and harmonics not to study the physical world but to develop logical thinking and promote reasoning. This can clearly be seen when he talks of astronomy.

'We shall therefore treat astronomy, like geometry, as setting us problems for solution,' I said, 'and ignore the visible heavens, if we want to make a genuine study of the subject and use it to convert the mind's natural intelligence to a useful purpose.' (Plato, 2011b, p. 289)

Plato uses mathematics as a tool to build the foundation for what he believes is the ultimate tool for seeking truth, The Dialectic.

The Dialectic (discourse)

The Dialectic or discourse is process of communication where two people attempt to discover the truth of a matter through discussion, unlike rhetoric or debate where one person chooses a side or the focus is to argue a specific side. The Dialect seeks to find a mutual agreement on truth. Plato believes that the Dialectic is the only way to find truth.

> So when one tries to get at what each thing is in itself by the exercise of dialectic, relying on reason without any aid from the senses, and refuses to give up until one has grasped by pure thought what the good is in itself, one is at the summit of the intellectual realm, as the man who looked at the sun was of the visual realm.
> 'And should we add it is only the power of dialectic that can reveal it, and then only to someone experienced the studies we have just described? There is no other way, is there?'
> 'We can claim that with certainty.'
> 'Well, at any rate no one can deny that it is some further procedure (over and above whose we have been describing) which sets out systematically to determine what each thing essentially is in itself.' (Plato, 2011b, p. 292- 293)

Plato states further that the Dialectic is only strengthened though the study of mathematics (Plato, 2011b). Plato would assert that the Dialectic of itself is the both the foundation and the pinnacle representation of human intelligence.

> 'Then you agree that dialectic is the coping-stone that tops our educational system: it completes the course of studies and there is no other study that can rightly be placed above it.' (Plato, 2011b, p. 295)

> 'Dialectic, in fact, is the only procedure which-proceeds by the destruction of assumptions to the very first principle, so as to give itself a firm base. When the eye of the mind gets really bogged down in a morass of ignorance, dialectic gently pulls it out and leads it up, using the studies we have described to help it in the process of conversion.' (Plato, 2011b, p. 294)

If we take Plato's assertion as truth then we can infer what Plato might have thought about the language of math and what fluency means.

Descartes on Method, Deductive reasoning, and correct learning.

Rene Descartes, a philosopher and mathematician, is best known as the father of modern philosophy. Descartes's is known in the field of mathematics as the inventor of analytical geometry, and the Cartesian coordinate system. Through his writings, *Rules for the guidance of our native powers*, and *Methodology*, I will unpack his philosophy concerning methodology, deductive reasoning, and the correct learning process, and their applications towards fluency, and mathematics as a language.

The political situation in the mid-17th century was not friendly to modern scientific or philosophical thought. Descartes writings occurred around the time of Galileo's execution for heresy. These writings reflect this reluctance to commit to open debate on his philosophy or method.

Descartes focused on method and relied strictly on deductive reasoning in his studies. Descartes method became the model for the geometric proofs we use today. Descartes based his philosophy around the methods used in his studies and proofs.

Descartes was extremely focused on how we obtained the truth, to him the method or way you found truth was almost as important as its validity. Descartes states, "In the search for the truth of things method is indispensible" (Descartes, 1958, p. 13); it was this method that profoundly shaped his philosophy.

This method was based on ordering our studies, breaking each down to its simplest form and working from the simple to the most complex (Descartes, 1958).

> The secret of this whole method is, therefore this: that in all things we carefully take note of that which is most completely absolute. Secondly we must note that the pure and simple nature which we are in position to intuit *primo et per se*, i.e., as not [in our knowing of them] dependent on any others, but as immediately disclosed to us either in this and that sense-experience, or by a light that is native in us, are few in number; and as we have been saying, it is these which should be carefully observed; for they are those natures which we have spoken of as being the simplest in each of the series (Descartes, 1958, p. 24-25).

Though this ordering of study Descartes hoped to isolate problematic areas and reduce errors, for like Plato, Descartes believed that it was the short comings of our physical natures that error was introduced.

Descartes believed that through method it was possible to reduce errors caused by the human condition. Descartes like Plato believed that it was the short comings of our physical natures caused us to be unable to find the truth of a matter; however Descartes did not believe that we should separate ourselves from our humanity or try to control it through abstaining from human behavior.

> Good sense is of all things in the world the most equitably distributed; for everyone thinks himself so amply provided with it that even those most difficult to please in everything else do not commonly desire more of it than they already have. It is not likely that in this respect we are all of us deceived; it is rather to be taken as testifying that the power of judging well and of distinguishing between the true and false, which, properly speaking , is what is called good sense, or reason, is by nature equal in all men; and that the diversity of our opinions is not due to some men being endowed with larger share of reason than others, but solely to this, that our thoughts proceed along different paths, and that we are therefore, not attending to the same things. For to be possessed of good mental powers is not of itself enough; what is all-important is that we employ them rightly. The greatest minds, capable as they are of the greatest virtues, are also capable of the greatest vices; and those who proceed very slowly may make much greater progress, provided they keep to the straight road, than those who, while they run, digress from it. (Descartes, 1958, p. 93)

Descartes used the methodology to reduce the impact of those behaviors on his research, requiring his proofs to be free of opinion and assumption (Descartes, 1958).

Descartes identifies how and what type of truths we should commit our minds to advancement. These include ideas of how to address new topics, and to what end we should study truths. Descartes and Plato agreed on the purpose of education and learning being self improvement or the attainment of knowledge. Descartes states, "The aim of our studies should be that of so guiding our mental powers that they are made capable of passing sound and true judgments on all that presents itself. (1958, p.1)

Descartes believed that everything was within the capability of human discernment, but unlike Plato Descartes states that not all problems or questions can be answered with the knowledge at hand.

> For, be it noted, no questions are to be taken as being perfectly understood, save those in which we apprehend distinctly the three prerequisites: (1) what the marks are that enable us to recognize what we are seeking when we come upon it; (2) from what precisely we ought to deduce this; and (3)the manner in which these two [the data and the conclusion to which they lead] are proved so to depend each on the other that it is impossible for either to be changed in any respect while the other remains unchanged (Descartes, 1958, p. 69).

Descartes limits the questions because he believes that our conscious awareness is only capable of thinking in the immediate, therefore we are limited by what we know in the present (Descartes, 1958). Therefore we should not commit time and resources to the study of abstract theories that cannot be linked to a discernable truth.

> Only those objects should engage our attention, to the sure and indubitable knowledge of which our native powers seem adequate. We reject all modes of knowledge that are merely probable... and in respect of which doubt in not possible (Descartes, 1958, p.4).

> We should not occupy ourselves with that which we are unable to have certain attitudes equal to that of arithmetical and geometrical demonstrations (Descartes, 1958, p.7).

It is for this reason that Descartes chooses deductive reasoning over inductive logic for his method.

Deductive Reasoning

Descartes' method is based on deductive reasoning; Descartes felt that inductive reasoning was to open ended and left open to errors because it was based on erroneous assumptions. "When investigating we should not examine what others have opined or, what we ourselves conjecture, but what we can clearly evidently intuit or can deduce with certainty" (Descartes, 1958, p. 8). Descartes believed that all truths were related and therefore to understand the truth of a thing or matter it was necessary to trace that truth from something that was already known. Descartes states that we should start from the simplest known truth and move to the more complex with each step based on a connected truth.

> That we start from what is so simple and evident as to be indubitable; and that in advancing from the simple to the complex no step be taken which is not similarly indubitable (Descartes, 1958, p. xii).

> For the distinguishing of the simplest things from those that are complex, and in the arranging of them in order, we require to note, in each and every series of things in which we directly deduce truths from other truths, which thing is simplest, and then to note how all the others stand at greater or lesser or equal distance from it (Descartes, 1958, p. 23).

Descartes states that it is these steps or connections that we can conduct an examination for the truth with certainty.

Fluency

Descartes realized that there is some level of proficiency required to advance in mathematics and science and that there would be some that would quite either because they became bored or due to the perceived complexity they encountered.

> I am not surprised that many people, even among the talented and learned, on sampling these sciences, very soon set them aside as being idle and puerile; or else, judging them to be exceedingly difficult and intricate, they have stopped short at the very threshold (Descartes, 1958, p. 17).

> The great majority of men on finding the cause of a thing to be quite perspicuous and simple consider that they are learning nothing.
> Everyone ought, therefore, to accustom himself to grasp in thought things so simple, and at any one moment so few, that he will never thereafter be tempted to think that he is knowing anything, save when he has an intuition of it no less distinct than the intuition he has of that which he knows most distinctly of all. Some are indeed born with much greater aptitude than others for such intuitive discernment. But by art and exercise our [native] mental powers can be

> immensely improved. The point upon which, as it seems to me, I ought to insist above all others is therefore this: that everyone should confirm in himself the conviction that it is not from things lofty and obscure, but solely from what is easy and readily accessible, that sciences, however recondite, have to be deduced (Descartes, 1958, p. 40).

Descartes developed procedures and rules to facilitate with these problems. These rules sought to reduce the perceived complexity by breaking them down into their simplest parts.

> For my part, conscious as I am how slender are my powers, I have resolved, in my search after knowledge of things, perseveringly to follow such an order as will require that I begin always with the things which are simplest and easiest, and that I never step beyond them until in their regard there remains, it would seem, nothing more to be done (Descartes, 1958, p. 20).

> When approaching a problem or question that seems to be beyond our understanding we should seek to divide it into those parts we understand and can prove and those that we cannot.
> Secondly, we have to deal with the things themselves, through only in so far as they can come within the reach of the understanding. So taken, we divide them into those natures which are completely simple and those which are complex or composite (Descartes, 1958, p. 37).

> We ought [for the training of the mind in perspicacity] to concentrate our native powers on those things which are simplest and easiest, and to dwell on them at such length that we thereby confirm ourselves in the habit of intuiting truth distinctly and perspicuously (Descartes, 1958, p. 39).

The rules helped to limit the study to more manageable chunks. This however leaves a large amount of truths and made it difficult to find the connections between them. It is to this end that Descartes consistently promotes a kind of fluency or fluidity of thought.

Descartes advocated a thorough study of a specific topic until all questions were answered and nothing more could be obtained by further study (Descartes, 1958). He also promoted practice and rehearsal to increase the speed and understanding.

> For the completing of our knowledge, the things which bear on what we have in view must one and all be surveyed by a movement of thought which is continuous

> and nowhere interrupted, and embraced in an enumeration which is sufficient and orderly (Descartes, 1958, p. 27).

> Thus if I have found, by way of separate operations, what the relations is, first, between the magnitudes A and B, then between B and C, and finally between D and E, I do not, in so doing thereby see what is the relation between A and E, nor am I able to learn of it from the truths antecedently known unless I recall all of them. This is why I have to run them over several times, the imagination operating with a motion so continuous, that while it is intuiting each step it is simultaneously passing go on to the next, until I have learned to pass from the first to the last so quickly, that almost none of the steps are left to the care of memory, and that it then seems as if I were intuiting the series simultaneously as a whole. And not only is the memory thus strengthened, the sluggishness of our mental powers is diminished and their capacity extended (Descartes, 1958, p.27).

Descartes added a further limiting factor, when we encounter an area where we cannot seem to go forward with this fluidity of thought we should not advance until such time as we can intuit sufficiently well (Descartes, 1958). Descartes believes that it is important for us to communicate but not necessarily in discourse.

Descartes believes that we should learn the truths and prove them to ourselves (Descartes, 1958). He encourages written communication not for the purpose of communication but to help reveal short comings and promote further learning of others.

> Do we not always give closer attention to what we believe will be read by others than to what is written only for ourselves? How often what has seemed true to me when first thought of has seemed false on my attempting to commit it to writing. Everyone is indeed under obligation, in proportion to his abilities, to promote the good of others; to be of service to no one is indeed to be worthless (Descartes, 1958, p. 134).

Descartes reluctance to participate in discourse was motivated mainly by the political climate in which he found himself. Descartes commitment to written proof gave us the modern Geometric proofs we use today (Descartes, 1958). These written add an additional dimension to what we may call fluency in mathematics.

Analysis

The current definition of fluency in mathematics is in conflict with the ideas of fluency in language. Since mathematics can be considered to be a meta-language. This definition is lacking in scope. The definition of fluency in mathematics is the ability to perform calculations quickly and accurately. To explore this further and define a more accurate definition of fluency in mathematics we need to identify what fluency really means in linguistics. Using Halliday a noted linguist we unpack fluency and determine the key characteristics of fluency and compare them with the philosophies of the original authors of the language of mathematics, Plato and Descartes.

Halliday

Halliday's ideas surrounding fluency in language are defined first by how he defines language and its role in learning and secondly the way we learn a language. Each of these areas have significant implications how he interprets fluency. The way Halliday defines language is the base upon which he builds his philosophy on learning.

Halliday defines language as a system of meaning, more to the point a semiotic system of meaning (Halliday, 2003). This semiotic system, (system of meaning), has the capability to communicate and create meaning. This idea has a broad effect; it implies that all learning is done as a function of language. Halliday believes that we use language to describe the physical and theoretical world around us (Halliday, 2004). "Most obviously perhaps, when we watch small children interacting with objects around them we can see that they are using language to construe a theoretical model of their experience" (Halliday, 2004, p. 15); this idea has great implications when we view mathematics as a language.

We commonly use mathematics to describe the physical and theoretical world around us today. Terms may seem common but without the use of mathematics those terms would not be in existence today. Circle, sphere, and squares would not be terms used today without mathematics. These terms are defined solely through the use of mathematics. Many of the terms we use to describe everyday activities have their roots in mathematics and have become so ingrained in the everyday usage that they seem to be of the mother tongue. We learn these through sight recognition and simple games as children. This leads us to Halliday's three facets of language learning.

Halliday proposes that there are three facets of language learning; learning language, learning through language, and learning about language (Halliday, 2004). These facets may be individually or simultaneously as we learn new systems of language. Halliday explains how children learn language using these facets.

The first facet, learning language is characterized by the use of "protolanguage" (Halliday, 2004). This "protolanguage" consists of simple gestures and sounds but is incapable of creating meaning.

> Protolanguage has a semantics and a phonology, but no level of grammar between the two. In other words, it is not yet stratified. The grammar emerges later, as the child moves from child tongue to mother tongue during the course of the second year of life (Halliday, 2003, p.13).

This language has a simple semantics but no grammar to facilitate the creation of meaning. In mathematics this might be seen when children communicate more or less, different and the same. When children begin to create meaning and explore the hypothetical they are moving to the second facet of language learning; learning through language (Halliday, 2003).

Children begin to learn through language when they are able to define or create new meanings by giving context through the use of grammar (Halliday, 2004). This is the first step where children are able to construe the world around them through the language they have learned. This does not mean that a child can determine what is hot and cold, but the child is able to describe vocally the why or what something is based on previous knowledge and the use of grammar to make logical sense of the description. This use of grammar allows children to create meaning on a hypothetical level as well (Halliday, 2004). In mathematics this would be the ability to understand that something has more because it is 3 units larger than the original or to explain that if something would be smaller if it was three units less in measurement. The third facet of learning language is reached when children understand how to create meaning through language.

When students understand the function of grammar and are able to manipulate the language to create meaning is the third facet or learning about language. "'Learning about language'; in other words, coming to understand the nature and functions of language itself" (Halliday, 2004, p. 332). This facet represents fluency in a language. The ability to understand why grammar changes meaning and how to manipulate it to create new meaning, allows for the exploration of the hypothetical world or the abstract. This may be shown by the use of creative speech or writing styles, to be able to put the language in a form where it may be reflected on (Halliday, 2004). In mathematic it would be the ability to understand and manipulate functions using reciprocal operations, or being able to identify a function by its graph and manipulate its shape. This leads us to a more accurate description of what Halliday would consider fluency in the language of mathematics.

Halliday states that fluency begins during the transition from the second facet of language learning; "learning through language" to that of "learning about language" (Halliday, 2004). Halliday indicates that the traits of fluency are the ability to communicate abstract theorization, and an understanding of how to use grammar to develop meaning (Halliday, 2004). Halliday emphasizes that the focus of fluency should be on discourse the ability to communicate rather than on the form of the language (Halliday, 2004).

Plato

Plato would describe fluency based on his beliefs of learning, inductive logic, and the dialectic. Plato's philosophy is based on the idea that learning is recollection, and that that the search for truth should only be done through reasoning alone (Plato, 2011a). These factors influence is adoption of inductive logic, and his love of the dialectic. The pursuit of previously learned knowledge is the driving force of Plato.

The Platonic belief that all learning is recollection of knowledge learned in a previous life time plays the central role in Plato's philosophy. "All Knowledge is recollection based on previous experience 'what the soul has learned' (Plato, 2011a, p. 6). Plato does not believe that we ever really "learn" anything new or for the first time, we just remember that which we already knew. This seems simple at first glance but the argument is cyclical in nature, in essence we must have known everything at one time, since we can only recall what we already learned; there is no opportunity for further learning in Plato's belief system. The idea that learning is only the recollection of previously learned knowledge is partially true. We must be able to apply what we already know to new situations, the old knowledge providing the base upon which new learning can occur. This idea of recollection leads Plato to the belief that the truth can only be found through the use of reasoning only.

The use of reasoning only as a means of obtaining truth, ignoring the physical world and emphasizing the use of high order, or abstract thinking, is the trademark of Platonic thinking.

> "Now how about the acquisition of wisdom? Is the body a hindrance, or is it not, if you use it as an accessory in the search? What I mean is, do sight and hearing provide men with any true knowledge, or are even the poets always trying to tell us something like this, that nothing that we hear or see is accurate? And yet if these bodily senses are not accurate or reliable, the others are hardly likely to be-for all the others are inferior, I suppose, to these. Don't you think so?"
> "I do indeed." He said.
> "When, then, does the soul attain to truth?" he went on. "when it tries to investigate anything with the help of the body, the body quite clearly deceives it....So it is only through reasoning, if at all, that any part of reality can be plainly understood" (Plato 2011a, p. 71).

Plato emphasized that we need to forgo or ignore the physical nature of our existence in order to obtain truth (Plato, 2011a). Although Plato ignores the physical nature of human beings he did not however ignore observations of the physical world around him. Plato used these observations to develop theoretical or hypothetical assumptions to explore using reasoning only. This shows that Plato required language to have the ability to explain the hypothetical or abstract. Plato held mathematics in high regard because it was used extensively to explore these theoretical ideas (Plato, 2011b). Plato's beliefs on learning and the use of reason lead him to the adoption of inductive logic.

Inductive logic is the use an observation or assumption and through the use of examples following a connected path arrives at a reasonable conclusion. This is a foundational argumentative style of Plato, although it relies on an initial observation it plays to his use of reason only as an argumentative style. This style of reasoning or logic requires the extensive use of abstract thought or theoretical thinking so loved by Plato. The importance of this style of thinking to mathematics as a language is its requirement for communication or discourse. This requirement for discourse leads us to Plato's crown jewel of education The Dialectic (Plato, 2011b).

The Dialectic is the two way communication or search for the truth through mutual agreement (Plato, 2011b).

> 'Dialectic, in fact, is the only procedure which-proceeds by the destruction of assumptions to the very first principle, so as to give itself a firm base. When the eye of the mind gets really bogged down in a morass of ignorance, dialectic gently pulls it out and leads it up, using the studies we have described to help it in the process of conversion.' (Plato, 2011b, p. 294)

Unlike rhetoric or debate which seek to sway one side or another to a specific viewpoint, the dialectic seeks to use discussion to obtain a mutually agreed upon truth based on logic (Plato, 2011b). The importance of this when speaking of mathematics as a language is its requirement for two way communications, this implies a similar level of fluency for all participants.

Fluency according to Plato should then include the elements of recollection, high order reasoning, and communication. Platonic fluency would require the ability to discus abstract or theoretical ideas to discover or recall previously learned truths.

Rene Descartes

Rene Descartes's description of fluency would be most influenced by his philosophy of method, deductive reasoning, and his own views on the importance of fluency. Although Descartes was the only author to directly address fluency it is necessary to understand the impact of his philosophy to understand why and how he addresses it. Descartes's philosophy like Plato's is based on the idea that the human emotion and desires can cause errors of thought (Descartes, 1958). Descartes however did not reject the physical nature of human existence; he adopted method as a means to guard against error.

> Good sense is of all things in the world the most equitably distributed; for everyone thinks himself so amply provided with it that even those most difficult to please in everything else do not commonly desire more of it than they already have. It is not likely that in this respect we are all of us deceived; it is rather to be taken as testifying that the power of judging well and of distinguishing between the true and false, which, properly speaking , is what is called good sense, or reason, is by nature equal in all men; and that the diversity of our opinions is not due to some men being endowed with larger share of reason than others, but solely to this, that our thoughts proceed along different paths, and that we are therefore, not attending to the same things. For to be possessed of good mental powers is not of itself enough; what is all-important is that we employ them rightly. The greatest minds, capable as they are of the greatest virtues, are also capable of the greatest vices; and those who proceed very slowly may make much greater progress, provided they keep to the straight road, than those who, while they run, digress from it. (Descartes, 1958, p. 93)

Descartes believed that the method used to find truth was almost as important as its validity (Descartes, 1958). This philosophy drove him to create rules, which he strictly followed. These rules served to isolate and insulate his studies from human emotion and desires which he believed were the chief cause of error (Descartes, 1958).

The method Descartes used was designed reduced each element to their smallest component. Descartes was of the mind that many errors were caused by the scope of a question and preferred to reduce them to their smallest form. Descartes stated "That we start from what is so simple and evident as to be indubitable; and that in advancing from the simple to the complex no step be taken which is not similarly indubitable" (Descartes, 1958, p.xii). This implies that we should focus our studies on what we know to be true and then expand to the next level of complexity only when all the sub-elements have been thoroughly explored. This method lead Descartes to prefer deductive reasoning over inductive.

Descartes's use of deductive reasoning is based on his believe that we should work from what we know not from assumptions. Descartes believed that inductive reasoning was based on assumptions and therefore had inherent capacity for error (Descartes, 1958). Descartes's search for truth was based on building from what we know.

> For, be it noted, no questions are to be taken as being perfectly understood, save those in which we apprehend distinctly the three prerequisites: (1) what the marks are that enable us to recognize what we are seeking when we come upon it; (2) from what precisely we ought to deduce this; and (3)the manner in which these two [the data and the conclusion to which they lead] are proved so to depend each on the other that it is impossible for either to be changed in any respect while the other remains unchanged (Descartes, 1958, p. 69).

This implies that the language of mathematics should be based on a building block process building new knowledge/meaning from what we already know to be true. Descartes idea of fluency is influenced by both method and deductive reasoning.

Descartes determined fluency to be the ability to connect truths together seamlessly, in a deductive manner without thought. Descartes used the term "intuit" as a means of showing this seamless thought process. Descartes reiterates this throughout his method, "We ought [for the training of the mind in perspicacity] to concentrate our native powers on those things which are simplest and easiest, and to dwell on them at such length that we thereby confirm ourselves in the habit of intuiting truth distinctly and perspicuously" (Descartes, 1958, p. 39). It was his belief that through practice we could work through a series of deductions repeatedly and at some point we would understand all the steps as truth and therefore we would "intuit" it as truth (Descartes, 1958). Descartes however extends fluency into that of written communication.

Descartes did participate so much in discussions but he did write letters in defense of his philosophy and studies. Descartes believed more in the use of written debate or rhetoric to defend his ideas. Descartes believed that we should be able to communicate in the written form for two reasons; first to help identify errors, and secondly to better our fellow man.

> Do we not always give closer attention to what we believe will be read by others than to what is written only for ourselves? How often what has seemed true to me when first thought of has seemed false on my attempting to commit it to writing. Everyone is indeed under obligation, in proportion to his abilities, to promote the good of others; to be of service to no one is indeed to be worthless (Descartes, 1958, p. 134)

This level of fluency requires an intimate knowledge of grammar and a comprehensive understanding of the subject matter. Descartes has by far the most demanding requirements of fluency, requiring the ability to build meaning using grammar and previous knowledge and to communicate not only verbally but in written form.

What does Fluency mean (Halliday, Plato, and Descartes)

From the writings of Halliday, Plato and Descartes we can conclude that there is agreement on three specific ideas which surround the definition of Fluency. The first being fluency must have the ability to communicate the abstract or higher consciousness. The second, fluency must be able to create meaning through the applied use of grammar. The third, fluency must be able to construct meaning from prior learned knowledge. These ideas remain intact within each writer's philosophy and methods.

There is some contention between the authors and the idea of what the role of discourse is in learning. Descartes, being more focused on individual self learning than on a more social setting, disagrees stating that it is more important to use written communications and rhetoric, mainly for the purposes of identifying mistakes or omissions in the respective proof, and secondarily for the betterment of mankind (Descartes, 1958). Plato's idea that it is through the two way communication of the dialectic that we discover truth (Plato, 2011b). Although they disagree on how communication is done there is no disagreement on the concept that ideas should be discussed.

If we use the ideas common to all three authors we can develop a more comprehensive picture of what fluency in the language of mathematics looks like. Fluency in mathematics should include all four factors. Fluency is the ability to create new meanings through the use of grammar and prior knowledge/meaning, to express abstract thought, and to communicate through discourse. This new definition of fluency is far different from the currently accepted definition, and consequently requires us to look at the way we teach mathematics.

Implications

Fluency in mathematics old: The ability to calculate quickly and accurately

Fluency in mathematics new: The ability to create new meanings through the use of grammar and prior knowledge/meaning, to express abstract thought, and to communicate through discourse.

The redefining of what fluency means in mathematics requires us to look more in-depth at the way we teach mathematics in the United States. The new definition of fluency in mathematics should lead us to look at mathematics education in four new ways. First we need to understand and the curriculum should promote discourse at an earlier age. Secondly this discourse should be standardized to use the same terminology across all age groups. Third the idea that memorization in mathematics is just busy work and that doing repetitive calculations is just busy work. Fourth the impact fluency has on mathematics avoidance and anxiety. Fluency needs to be taught at an early age and the earlier the better.

The curriculum of mathematics should focus more on developing dialogue and discussion using the language of mathematics at an early age. Elementary students spend less time working with mathematics and science than any other subject. The understanding that mathematics is a language and that students learn new languages best at an early age has not been capitalized on in the United States (Stevenson, Lee, & Stigler, 1986). The average student enters junior high school with only a basic conceptual knowledge of mathematics with little or no fluency. The importance of early exposure to the language of mathematics is to developing meanings and a "protolanguage" that students can build on. The education process needs to promote the use of language of mathematics whenever possible using the correct meanings and terminology.

The correct terminology and meanings should be used throughout a child's education to allow for growth in the language. Students should learn to use mathematics to define other mathematical terms, often this is not the case. Teachers often instruct students to write the definition in their own words, this leads to confusion and the inability to construct new meanings using the language of mathematics. Teachers and text books often show short cuts using differing terminology than that of the excepted (Halliday, 1996). The term cross multiplication is a common example of this kind of improper terminology. This results in the learning of a process but creating incorrect meanings. The goal of the classroom should be centered on getting children to think and speak mathematically correctly not always about getting the correct answer the first time (Halliday, 2004). Repetition is not always a bad thing when learning.

The learning through repetition is a tool that is necessary when learning a new language and the language of mathematics should not be taught any different. The common use of repetition in mathematics as a learning process has been seen culturally in the United States as a bad thing (Stodolsky, 1985). This view seems somewhat short sighted as it is a common tool used in all language learning activities. Common activities we see in language learning include: sight word lists, spelling tests, definitions, and conjugation lists. The memorization of these lists could be viewed as busy work as well but when coupled with everyday use they are not. These activities are done to develop fluency and accuracy in the language. The advent of the calculator and the function which they are capable makes the memorization of some processes and information tedious, at the same time the calculator makes some material more accessible. The calculator cannot teach the mind to recognize patterns, and concepts. This repetition is necessary just as it is in language to build fluency and speed up the thinking process (Descartes, 1958). Repetition is necessary to develop fluency and this fluency may provide the key to combating Mathematics Avoidance and Anxiety.

The concept that fluency in mathematics implies some level of communications skills that requires discourse allows us to understand how students perceive mathematics, and why they may view it with some trepidation. Fluency with discourse as one of its key elements brings us back to the idea that the way we teach mathematics may be one of the reasons there is a prevalence of mathematics avoidance in the United States. There are two reasons this may have such a large impact on students in the first being the number of English as a second language students in schools, and the second the way instruction in the united states does not promote discourse in mathematics at an early age. These two reasons may require some change in the way we approach mathematics at the elementary school level.

The larger number of students that do not speak English as a primary language may be one reason why mathematics avoidance is so prevalent in the United States. Halliday states that students that encounter this type of meta-language that is not rooted in their "mother tongue," find it extraordinarily difficult. The language of mathematics is full of special probabilities and does not conform to conversational grammar usages (Halliday, 1996). This may be an area that needs more research and a change in the way students are placed in mathematics classes. This may be further compounded by the lack of mathematics discourse found at the elementary level of education.

Students in the United States are not required to have any level of fluency until they reach middle school or junior high school, this may be a significant cause of mathematics avoidance. Students enter elementary school using the equivalent of protolanguage in regards to mathematics, and are expected to have some fluency when they leave. The use of short cuts and non standard mathematic terminology do not help this situation. There should be some study done to evaluate the amount of discourse used in elementary schools and an effort made to increase the amount of discourse in mathematics done at an earlier age. We require students to communicate effectively writing and conducting presentations for all other aspects of learning in elementary school grades but not in mathematics.

Conclusion

Mathematics avoidance seems to be a systemic problem in the United States through this research I explored the possible connections between mathematics avoidance and fluency. Mathematics may not be a formal language but a meta-language with different grammatical constructions that are significantly different from the common conversational language.

The current definition of fluency in mathematics only requires speed and accuracy in calculation. This definition is shallow and does not account for the multitude of roles fluency plays in learning. Through this study we have redefined fluency in mathematics to mean the ability to create new meanings through the use of grammar and prior knowledge/meaning, to express abstract thought, and to communicate through discourse. This definition more accurately fits the role that language and fluency play in learning.

The new definition lets us understand the role fluency plays in the problem of mathematics avoidance. Fluency has a direct effect on the difficulties that second language learners encounter when confronted with a technical language like mathematics. Students are required to have some fluency the middle and junior high level is another reason to take a more in depth look at fluency in mathematics.

This implies that changes are necessary to address lack of fluency at the middle and junior high school levels. These changes include the implementation of discourse in the language of mathematics, using correct terminology, and the role of homework in mathematics.

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